



Lecture12 Single-qubit gates and discrete quantum tomography

- Single-qubit gate
- discrete quantum tomography

quantum information

Superposition
entanglement
(Superposition of product states)

processor
memory (register)

1 qubit = $\begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$ Superposition $\rightarrow \cos(\frac{\theta}{2})|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})|1\rangle$

1 classical bit $\begin{cases} |0\rangle \rightarrow R \\ |1\rangle \rightarrow L \end{cases}$

$\theta = \frac{\pi}{2}, \phi = \frac{\pi}{2} \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \rightarrow R$

$\theta = \frac{\pi}{2}, \phi = \frac{3\pi}{4} \rightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})|1\rangle$
 $= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}(1+i)|1\rangle \rightarrow L$

2 qubit $\begin{cases} |00\rangle \rightarrow 0 \\ |01\rangle \rightarrow 1 \\ |10\rangle \rightarrow 2 \\ |11\rangle \rightarrow 3 \end{cases} \rightarrow \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |11\rangle) =$

unitary gate = transformation $\rightarrow$ total probability remains 1

$\hat{U}|\psi\rangle = |\psi'\rangle \rightarrow \hat{U}^{-1}|\psi'\rangle = |\psi\rangle$
 $\rightarrow \hat{U}\hat{U}^\dagger = \hat{I} = \hat{U}^\dagger\hat{U}$
 $\rightarrow \hat{U}^{-1} = \hat{U}^\dagger$

irreversible

for

for

for

for





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NMR
 $\frac{d\vec{M}}{dt} = \vec{M} \times \vec{B}$
 $\rightarrow \frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} = \vec{S} \times (-\vec{\Omega})$
 $\vec{\Omega} = (\Omega, 0, 0) \rightarrow \vec{\Omega} = (\Omega \cos \phi, \Omega \sin \phi, 0)$

Rotation Transformation

(4)

universal gate set
 $\{X, \text{PI}/8, H, \text{CNOT}\}$

X gate
 $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 $|0\rangle \rightarrow |1\rangle$ (NOT)

PI/8 gate
 $\begin{pmatrix} 0 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

Hadamard gate
 $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
 $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\phi_0 = \frac{\pi}{2} + \phi$
 $\tau = \frac{\pi}{\Omega \sin \phi}$

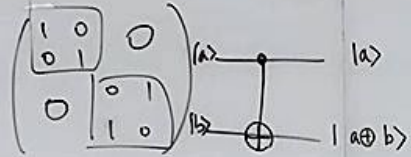
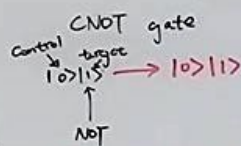
$\phi_0 = 0 \rightarrow \vec{\Omega} = (\Omega, 0, 0)$

$\theta = \frac{\pi}{2}, \tau = \frac{T}{2} = \frac{\pi}{\Omega}$
 $\theta = \frac{\pi}{4}, \tau = \frac{T}{4} = \frac{\pi}{\Omega}$



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$$\begin{aligned}
 & \begin{pmatrix} 1 \\ 2 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 2 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \\
 &= \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}
 \end{aligned}$$

